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2000 J. Phys. A: Math. Gen. 33 5157

(http://iopscience.iop.org/0305-4470/33/28/401)

J. Phys. A: Math. Gen. **33** (2000) 5157–5158. Printed in the UK PII: S0305-4470(00)09750-X

COMMENT

Comment on 'Exact solutions of the two-dimensional Burgers equation'

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Received 22 November 1999

Abstract. We demonstrate that an analysis in the paper by Sirendaoreji (1999 *J. Phys. A: Math. Gen.* **32** 6897–900) can be considerably simplified, and show that some of the derived expressions which were claimed to be solutions of the two-dimensional Burgers equation do not, in fact, satisfy that equation.

In [1] Sirendaoreji constructs solutions to the two-dimensional Burgers equation

$$(u_t + uu_x - u_{xx})_x + u_{yy} = 0. \tag{1.1}$$

After starting with a fairly general ansatz the author quickly reduces it to

$$u(x, y, t) = -k\left[1 + \tanh\frac{kx + R(y, t) - \ln P(y, t)}{2}\right] + B \tag{2.11}$$

with the following equations for the unknown functions R and P:

$$\begin{cases} k^2BP - k^3P - kP_t + kPR_t - 2P_yR_y + PR_y^2 + P_{yy} - PR_{yy} = 0 \\ k^2BP^2 - k^3P^2 - kPP_t + kP^2R_t + 2P_y^2 - 2PP_yR_y + P^2R_y^2 - PP_{yy} + P^2R_{yy} = 0. \end{cases} \tag{2.10}$$

It is obvious from (2.11) of [1] that only the combination $R(y, t) - \ln P(y, t)$ counts. This becomes clear when one multiplies the first equation of (2.10) with P and subtracts it from the second; the resulting equation can be written as

$$[R(y, t) - \ln P(y, t)]_{yy} = 0$$

with the solution

$$R(y, t) = \ln P(y, t) + ya(t) + b(t).$$

What remains from (2.10) of [1] is then

$$a(t)^2 + kya(t)' + kb(t)' + (B - k)k^2 = 0$$

and since a and b do not depend on y we find the solution

$$a(t) = \alpha$$
$$b(t) = t[-\alpha^2/k - (B - k)k] + \beta$$

where α and β are the integration constants. The result is therefore nothing more than the standard travelling wave of the kink variety

$$u(x, y, t) = -k\,[1 + \tanh((kx + \alpha y + \omega t + \beta)/2)] + B \tag{K}$$

with the expected dispersion relation

$$(B - k)k^2 + k\omega + \alpha^2 = 0. \tag{DR}$$

The solution (2.13) of [1] is the solution (K), (DR) above, but solutions (2.14) and (2.,17) of [1] are just their special cases. On the other hand, the proposed results (2.15) and (2.19) are *not* solutions of the original equation: the given R, P pairs (with nonlinear y-dependence) in fact only satisfy the *second* equation of (2.10), but *not* the first.

Reference

[1] Sirendaoreji 1999 Exact solutions of the two-dimensional Burgers equation *J. Phys. A.: Math. Gen.* **32** 6897–900